Efficient contests

Christian Riis

Norwegian School of Management

Nydalsveien 37

N-0442 Oslo

Norway

christian.riis@bi.no

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Abstract

In their seminal contribution, Lazear and Rosen (1981) show that wages based upon rank induce the same efficient effort as incentive-based reward schemes. They also show that this equivalence result is not robust toward heterogeneity in worker ability, as long as ability is private information, since it is not possible to structure contests to simultaneously satisfy self-selection constraints and first-best incentives.

This paper demonstrates that efficiency can be achieved by a simple modification of the prize scheme in a mixed (heterogenous) contest where contestants learn their type after entry. If contestants know their type before entering the contest, rent extraction becomes an issue. Implications for optimal contest design are also explored. Finally, the relationship between effort maximizing contests and profit maximizing contests are discussed.

Key words: Tournaments, Labor Contracts

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1 Introduction

Tournaments or contests are contracts that reward each agent according to his or her performance relative to that of others. The literature has thoroughly established that properly-designed contests have a number of attractive features. Lazear and Rosen (1981) show that, with risk-neutral agents, wages based upon rank induce the same efficient effort as piece rate schemes. Green and Stokey (1983) demonstrate that contests may dominate individual contracts if agents' outputs are due to common shocks. Malcolmsen (1984) points out that, since total prize expenses are fixed, tournaments reduce the principal's incentive to manipulate output measures, which may be a problem if payment is linked to a cardinal measure of performance. As a screening device, contests can be designed to identify and select the most appropriate candidate, an important feature of promotion decisions in organizations,[1] in procurements,[2] and in R&D contests.[3]

A major challenge in contest design is dealing with agent heterogeneity. Neck and neck competition triggers greater effort from the participants,

[1] See for example the discussion in Baker, Jensen and Murphy (1988), Gibbons (1997) and De Varo (2006).

[2] See Dalen, Moen and Riis (2006).

[3] Examples of contributions include Che and Gale (2003), Fullerton and McAfee (1999), Taylor (1995) and Wright (1983).

but this effect rapidly disappears if the difference in performance between contestants become too large. Even when performance is unobservable, expected asymmetries between contestants tend to undermine incentives, as purported underdogs often give in, and presumably superior players, realizing their opponents' weaknesses, exert less effort. Furthermore, with private information, contestants' beliefs about their opponents' abilities impact their effort incentives, leading to inefficient selection as a possible outcome. Since heterogeneity tends to make the contest less predictable, it may stimulate strategic behavior.[4]

These concerns have given rise to a number of contributions detailing how contest design may be modified in order to improve its efficiency in instances of agent heterogeneity. There are two main proposals: one is to sort contestants into subgroups or "leagues," each characterized by a high degree of homogeneity, and then arrange separate contests for each sub-group. The other proposal is to adjust the contest rules in a pooled contest in order to restore incentives. Neither approach is trivial. Lazear and Rosen (1981) show that self-sorting fails in their model since all agents prefer to participate in

[4]Bull, Schotter and Weigelt (1987), in their experimental comparison of piece rate schemes and tournaments, find systematically larger effort variance in tournaments than in piece rates. The authors' main explanation is that "a tournament, unlike the piece rate, is a game and so requires strategic, as opposed to simply maximizing, behavior."

the "high-ability" league. They also demonstrate that if contestants participate in the same "mixed" contest, information problems prevent the contest organizer from providing all participants with first-best incentives.

Still, both formal and informal sorting mechanisms are common in contests as well as in contest-like situations. For example, researchers competing for tenure have typically already undergone a process of self-sorting depending on the reputation of the school/institution to which they apply and the position involved. Hence groups of competing candidates often form a more homogenous group than the entire population of researchers. Authors of scientific papers typically choose between different journals as outlets for communicating their research, in competition with their peers for publication. In sports, athletes self-sort between the events in which they plan to compete. In procurement contests, prequalification rounds remove less promising suppliers, thereby creating a more homogenous group of contestants for the final round.

Such self-sorting mechanisms resemble contests, as reaching the final is in itself valuable. Competing for at tenure at a top university is more difficult than at a mediocre one, but it is also more prestigious. Prequalifying for a procurement contest is an honorable achievement for a firm. Fullerton and

McAffee (1999) formalize this point with a model in which the prequalification stage identifies and selects the two most promising firms; firms that eventually compete in the final closed R&D contest.[5]

If the number of contestants is large, and sorting mechanisms are effective, a high degree of contest homogeneity is achievable. However, with a limited number of potential contestants, the problem of heterogeneity is unavoidable.

If heterogeneous contestants are pooled in a mixed contest, contest rules can be adjusted in two ways. The first alternative is to modify the criteria used to identify a winner, a policy referred to as handicapping or affirmative action. The second option is to alter the prize structure by arranging the value of the prize according to the players' characteristics; for instance, by stipulating that the winner's prize be determined by the identity of the runner-up. That is the option explored in this paper. Clearly, these two alternatives are closely related in the sense that in any given contest, the expected marginal gain, which determines the incentive power of the scheme, is equivalent to the win probability multiplied by the winner's premium. Thus,

[5] An important insight in their paper is that the intitial prequalification cannot be arranged as a standard first-price or second-price auction. As the authors point out, the bidding strategy in auctions is for the bidder to evaluate her own bid as if she were the marginal bidder - however, the marginal bidder is the inferior contestant in the final round, and thus has zero value. To avoid this problem, the prequalification stage is arranged as an all pay auction with a monetary prize associated with reaching the final.

incentive power may be increased either by increasing the win probability, which is handicapping, or by increasing the winner's prize premium.

Handicapping has received a lot of attention in the literature. If the contest organizer has sufficient information about differences in contestants' capabilities, handicapping can be used as a device to align competition. See for example, the discussion in Che and Gale (2003) on R&D contests, Szymanski's (2003) survey on sporting contests, and Tsoulouhas, Knoeber and Agrawal's (2007) analysis of CEO contests. However, as McLaughlin (1988) points out, "The real problem with tournaments with heterogeneous contestants arises if the contestants' types cannot be identified."

The other alternative, to adjust the prize structure by conditioning the prize values on the identities of the winner and loser has received less attention in the literature. An example of such conditioning is found in organized chess. The winner of a chess game is rewarded with a greater rating improvement if she beats an opponent with a high rating than if she beats one with a low. This impacts the incentive structure in mixed chess tournaments. However, observability of distance measures is not an issue, since this is based on previous achievements.

Similar mechanisms have been discussed in the literature on patent design. Aghion, Harris, Howitt and Vickers (2001), in their analysis of step-by-step innovation, demonstrate that innovation incentives are greater if firms compete neck and neck, essentially, in contests with a high degree of homogeneity, than if the laggards fall behind technologically, which gives rise to heterogeneity. Based on this insight, Acemoglu and Akcigit (2006) raise the question of whether intellectual property rights should depend on the technological disparity between competitors. They show that the innovation rate is higher if the winner's prize, the degree of patent protection, is conditioned on technological disparity, which they refer to as "state dependent tournaments". Similar to the chess example, observability is not an issue here since technology generations are assumed observable.

Hopenhayn, Llobet and Mitchell (2006) explore a sequential innovation game in which the quality of firms' research ideas are heterogeneous. [6] They show that allowing innovators to select from a menu of patents that differ in scope, providing greater patent protection to better ideas, enhances efficiency. In other words, the prize obtained by the leading firm (the winner) depends on the specific innovation's incremental contribution, reflecting technological

[6]The model is not specified as a contest model, as each firm innovates only once with new firms arriving sequentially over time. However, the model can easily be reformulated as a contest model.

improvement on prior state of art technology. Distance is private information, but is revealed through the choice of "patent prize" from a menu.[7]

In this paper, I integrate the idea of conditional prizes in a contest model to derive the optimal design. As in contests, relative performance determines each agent's rank, but the values of the prizes depend on the distances between contestants as revealed by their choices from a *menu of prizes*. Lazear and Rosen (1981) present the basic model that I use. In their model the optimal design yields a first-best allocation. I demonstrate that efficiency is achievable by a simple modification of the prize scheme in a mixed (heterogeneous) contest.

I consider a model with one principal and two agents, each with ability drawn from a common distribution. Ability is private information. I maximize net revenue, the value of output net of prize expenses, conditional on agents' rationality constraints. The latter requires that agents be as least as well-off participating in the contest as they would in their best alternative choice. In the first version of the model I assume that agents learn their abilities after entering the contest, referred to as *ex ante* rationality constraints. Thus at the stage of entry, contestants are symmetric, which means

[7] Cornelli and Schankerman (1999) also introduce selection from a menu of patent lives and associated fees to provide more efficient R&D incentives.

that rent extraction is not an issue. Accordingly, the principal maximizes social surplus, and the optimal contest design is compatible with a first-best allocation.

In the second version of the model I assume that agents learn their abilities *before* entering the contest, known as *ex post* rationality constraints. Since high-ability agents thereby obtain an information rent, rent extraction becomes an issue, and the principal's optimal design yields an allocation that deviates from the social optimum.

In the contest literature, the principal's optimization problem is often specified in slightly different terms. A common specification is to assume that the contest organizer maximizes expected total effort, conditional on constraints on the prize structure. The relationship between the alternative approaches is discussed, and welfare implications drawn.

The rest of the paper is organized as follows. The first section presents the original Lasear and Rosen (1981) model, which is my starting point. The second section introduces heterogeneity and clarifies the design problem. The third section demonstrates that Lazear and Rosen's efficiency result can be generalized to heterogeneous contests. The fourth section explores various modifications regarding contest organizer's goals as well as constraints upon the optimization problem.

2 The Lazear-Rosen model

The model is specified as follows: Two contestants (j and k) of equal ability simultaneously invest μ_j and μ_k under strictly convex and symmetric investment cost functions $C(\mu)$. Their respective (lifetime) outputs are equivalent to investment plus a luck component, which is $q = \mu + \varepsilon$, assuming that ε has constant variance, zero mean, and zero correlation across contestants. Gross profit equals Vq; hence first-best allocation is represented by $V = C'(\mu)$.

The contestant with the largest output wins the contest and is paid the prize premium r in addition to the base wage W. Due to the luck component, the contestants' respective win probability functions are continuous in the two investment levels. If we let $G(\cdot)$ denote the CDF of the difference in luck terms $\varepsilon_j - \varepsilon_k$, and $g(\cdot)$ its density, the probability that j wins is $G(\mu_j - \mu_k)$, and that k wins, $1 - G(\mu_j - \mu_k)$. Thus, contestant $j's$ expected utility is:

$$W + G(\mu_j - \mu_k)r - C(\mu_j)$$

with first order condition,

$$g(\mu_j - \mu_k)r - C'(\mu_j) = 0$$

The equilibrium is symmetric, μ_j equals μ_k, and the first order condition can be expressed,

$$g(0)r - C'(\mu) = 0 \Longrightarrow \mu[g(0)r]$$

which determines a strictly increasing investment function $\mu[g(0)r]$.

Hence the prize premium r multiplied by the density of the noise term $g(0)$ corresponds to the incentive power of the classic piece rate reward scheme. The density of the noise term $g(0)$ is crucial. Due to the noise term, an increase in the level of investment will increase the player's win probability; the denser the noise distribution, the greater the increase. If the noise distribution is highly dense, pure strategy equilibrium breaks down, as demonstrated in Lazear and Rosen; see also Nalebuff and Stiglitz (1983). Observe that as the variance in the noise term approaches zero, the contest approaches an all-pay auction, in which equilibrium is found in mixed strategies.

Since $\mu[g(0)r]$ is strictly increasing, first-best allocation is achievable by a proper choice of prize premium. With efficient allocation, wealth is maximized, and correspondence to optimal piece rate schemes is established.

The next section introduces type heterogeneity.

3 A contest with type heterogeneity

Denote by θ the single agent's ability. I assume that ability is private information[8] and continuously distributed with symmetric probability density $f(\theta)$. With no loss of generalization I assume that the support is $[\underline{\theta}, \overline{\theta}]$. Since both type and effort are private information, the design problem is characterized by "dual information asymmetry."[9]

The investment cost function of an agent with ability θ is denoted by $C(\mu;\theta)$. I assume that investment cost as well as marginal investment cost $C'(\mu;\theta)$, are strictly decreasing in θ.

In first-best, the marginal value of investment V equals the agent's marginal cost:

$$V = C'(\mu;\theta) \Longrightarrow \mu^*(\theta) \tag{1}$$

which defines a strictly increasing first-best investment path $\mu^*(\theta)$.

[8] If ability were common knowledge, a set of handicaps adjusting for differences in incentive power due to the type distribution effect could be derived; see references above.

[9] A term introduced by Yun (1997).

As in L&R, first-best incentives are unattainable in standard contests. For clarification, consider a contest with base wage W and prize premium r, and assume the contrary principle that first-best incentives are achievable. The expected utility obtained by type θ_j, conditional on her opponent investing according to the first-best rule $\mu^*(\theta)$, can be expressed as,

$$W + \int_{\underline{\theta}}^{\overline{\theta}} G(\mu_j - \mu^*(\theta)) f(\theta) r d\theta - C(\mu_j; \theta_j)$$

with first order condition

$$\left[\int_{\underline{\theta}}^{\overline{\theta}} g(\mu_j - \mu^*(\theta)) f(\theta) d\theta \right] r = C'(\mu_j; \theta_j) \tag{2}$$

Under the assumption that the scheme is compatible with first best effort $\mu^*(\theta)$, the marginal utility, the left hand side of (2), equals the social marginal value V. Inserting $\mu^*(\theta)$ yields

$$\left[\int_{\underline{\theta}}^{\overline{\theta}} g(\mu^*(\theta_j) - \mu^*(\theta)) f(\theta) d\theta \right] r := \mathbf{f}(\theta_j) r = V \tag{3}$$

which holds if $\mathbf{f}(\theta_j)$ is constant. However, with some rare exceptions[10], $\mathbf{f}(\theta_j)$

[10] $\mathbf{f}(\theta)$ is constant in the special case where θ is uniformly distributed and $\mu(\theta)$ is linear.

is not constant, making the scheme incompatible with first-best (1). For instance, with a standard symmetric and single peaked noise-term $g(\cdot)$, the incentive power of the contest increases in type density around its own type θ_j[11].

From the literature on price discrimination, it is widely known that the principal may extract more surplus by introducing a menu of contracts from which participants self-select, for instance by allowing contestants to choose between different combinations of base wage W and prize premium r. This approach is appropriate if preferences satisfy "single crossing," in which case a menu can be designed such that each type θ chooses prize premium $r(\theta)$ compatible with first best. Denote by $r^*(\theta)$ the appropriate prize premium, from (3)

$$r^*(\theta) = \frac{V}{\mathbf{f}(\theta)} \tag{4}$$

Thus, first-best incentives require that each type chooses a contract that

[11] The smaller the spread in the density $g(\cdot)$ of the noise term ε, the more weight is attached to types in a small area around θ_j. For clarification of this specific property, observe that with type heterogeneity agents compete *locally* in the following sense: on the margin, the agent's gross benefit from a small increase in her investment "bid" is proportional to the density of types investing at that specific level - that is, the density of opponents of exactly her own type. Clearly, given her ability, she beats inferior types with ease and loses to superior types. Thus, facing an opponent of unknown ability, she invests *as if* her opponent were of her own type, taking into account the probability of this match.

provides her with a prize premium $r(\theta)$ inversely related to $\mathbf{f}(\theta)$, reflecting the density of types locally around her own type. Thus if a contestant expects to have few opponents of approximately her own type, she must be rewarded with a higher prize premium.

In our case, single crossing is highly restrictive as it requires that $r(\theta)$ is monotonically increasing. First, in order to separate types, a higher prize premium r must be combined with a lower fixed wage W. Second, trading off a low base wage for a high prize premium is certainly more attractive if the agent has a high win probability. Third, since first best effort is increasing in type, high ability contestants are in optimum more likely to win. Thus if a low type prefers the prize combination W_L, r_H to W_H, r_L, where $W_L < W_H$ and $r_L < r_H$, a high type certainly does the same. Hence a downward sloping segment of $r(\theta)$ is incompatible with single crossing. Since first best requires $\mathbf{f}(\theta)r(\theta) = V$, this implies that unless $\mathbf{f}(\theta)$ is monotonically decreasing, a simple contract menu approach fails. Clearly, a decreasing $\mathbf{f}(\theta)$ is incompatible with any standard single peaked ability distribution. The next section shows how the contest design can be modified to deal with this problem.

This paper is not the first to address this specific sorting problem in contests. O'Keeffe et al (1984), Bhattacharaya and Guasch (1988), and

Yun (1997) all address similar problems. However, their models are formulated such that they avoid the non-monotonicity problem described above. O'Keeffe et al (1984) consider a model of self-selection that divides contestants into separate high- and low-ability leagues, in which self-selection is achieved by increasing the prize spread in the high-ability league, which prevents the climbing of low-ability contestants, and reducing the prize spread of the low-ability-league, which prevents the slumming of high-ability contestants into the low-ability league. To restore effort incentives in the low-ability league, the degree of monitoring precision in this league must be increased, which in turn increases the marginal return from effort.

Bhattacharaya and Guasch (1988) show that efficiency is restored through self-selection among wage contracts, in which each contestant's performance is compared to the output of an agent who exhibits the *least* efficient investment level. I will comment below on the motivation behind this specific ranking mechanism.

Yun (1997), addressing the first part of the L&R inefficiency result related to self-selection into homogenous leagues, considers two-prize standard contests with multiple agents, in which the proportion of agents awarded the low prize is determined endogenously. He demonstrates that by varying what he refers to as the "penalizing rule," the proportion of low prizes in each contest, he can establish a sorting device by which each type will self-sort into her own league.

The sorting mechanism used in these contributions, premised upon the notion that high-ability contestants have a stronger preference for high prize premia than do low-ability contestants, cannot support full efficiency. The reason is that the mechanism is compatible with first-best incentives *only if* the incentive problem is *one-sided,* in the sense that the optimal incentive power is monotonically increasing in type. However, this assumption is not compatible with standard ability distributions.

In Bhattacharaya and Guasch (1988) the non-monotonicity problem is avoided by assuming that each contestant competes against a threshold represented by the investment level of a hypothetical agent with the lowest possible ability. Consequently, the density of the noise term is strictly decreasing, since each contestant's investment exceeds the threshold. To facilitate implementation, the threshold can be approximated by the performance of the historically lowest achiever. In O'Keeffe et al and in Yun, the problem is avoided by introducing new elements, for instance, by manipulating the noise term.

The following section formally characterizes the modified contest scheme in the two-player case, and briefly discusses generalizations to n players.

4 The generalized contest

My first model is based on the following time structure:

Stage 1: Agents enter and pay an entry fee to the principal

Stage 2: Agents learn of their abilities θ, independently drawn from a probability function $G(\theta)$

Stage 3: Agents simultaneously and independently choose prizes from a prize menu

Stage 4: Agents simultaneously and independently choose effort.

The objective behind the condition that contestants learn their abilities after entry is to maintain first-best effort as the optimal benchmark, thus focusing on the design problem discussed in L&R and subsequent literature. If contestants were to know their abilities in advance, a standard trade-off between rent extraction and efficiency would arise.[12] Since contestants are symmetric ex ante, the entry fee is determined such that the agents' respective participation constraints bind, and optimal design corresponds to

[12] This is discussed in the next section.

the first-best benchmark.

In this model it is convenient, as is common in the literature, to represent self-selection by letting the contestants report their own abilities. Formally, the reporting stage and contest stage go as follows: first contestants j and k report $\widehat{\theta}_j, \widehat{\theta}_k$. As in L&R, contestants compete in investment levels μ_j, μ_k. If j loses the contest she is paid the base wage $W(\widehat{\theta}_j)$, and if she wins she receives the additional prize premium $r(\widehat{\theta}_j, \widehat{\theta}_k)$, which depends on her own announcement $\widehat{\theta}_j$ as well as on her opponent's report $\widehat{\theta}_k$. Observe that $r(\widehat{\theta}_j, \widehat{\theta}_k)$ may differ from $r(\widehat{\theta}_k, \widehat{\theta}_j)$, the prize premium that k obtains if she is deemed the winner. Finally, contestant j does not know the content of $k's$ report, and vice versa. However, as final rewards depend on the agents' respective choices from the prize menu, their choices are revealed ex post.

Expected social surplus, per player, is the difference between the expected value of effort and expected cost:

$$S = \int_{\underline{\theta}}^{\overline{\theta}} \left[V\mu(\theta) - C(\mu(\theta), \theta) \right] f(\theta) d\theta$$

Expected profit can be written as,

$$\begin{aligned}\Pi &= S - U = \int_{\underline{\theta}}^{\overline{\theta}} [V\mu(\theta) - C(\mu(\theta),\theta) - u(\theta)]\, f(\theta)d\theta \\ &= \int_{\underline{\theta}}^{\overline{\theta}} [V\mu(\theta) - C(\mu(\theta),\theta)]\, f(\theta)d\theta - \int_{\underline{\theta}}^{\overline{\theta}} u(\theta)f(\theta)d\theta \end{aligned} \tag{5}$$

with rationality constraint

$$\int_{\underline{\theta}}^{\overline{\theta}} u(\theta)f(\theta)d\theta \geq \overline{u} \tag{6}$$

The constraint clearly binds: thus by inserting (6) in (5), expected profit can be expressed as,

$$\Pi = S - \overline{u} \tag{7}$$

The principal maximizes Π conditional on the participation constraint $\overline{u}$. It follows from (7) that the principal thus maximizes social surplus S, and so the optimal contest scheme yields a first-best outcome.

Contestant $j's$ utility as function of her type θ_j and report $\widehat{\theta}_j$ is expressed,

$$U(\theta_j, \widehat{\theta}_j) = \max_{\mu} \left[W_2(\widehat{\theta}_j) + \int_{\underline{\theta}}^{\overline{\theta}} G(\mu - \mu(\theta)) r(\widehat{\theta}_j, \theta) f(\theta)d\theta - C(\mu;\theta_j) \right] \tag{8}$$

in which $\mu(\theta_j)$ denotes the optimal investment for type θ_j measured in a truth-telling equilibrium. As the object function is strictly concave in μ, $\mu(\theta_j)$ is continuous and strictly increasing in θ_j.

Consider the sorting conditions. From the envelope theorem it follows that

$$\frac{dU(\theta_j,\widehat{\theta}_j)}{d\widehat{\theta}_j} = W_2'(\widehat{\theta}_j) + \int_{\underline{\theta}}^{\overline{\theta}} G(\mu(\theta_j) - \mu(\theta))r_1(\widehat{\theta}_j,\theta)f(\theta)d\theta = 0 \qquad (9)$$

Sorting requires that the marginal benefit of a higher announcement $\widehat{\theta}$ is monotonically increasing in type θ. Differentiating (9) with respect to type θ_j yields,

$$\frac{dU^2(\theta_j,\widehat{\theta}_j)}{d\widehat{\theta}_j d\theta_j} = \int_{\underline{\theta}}^{\overline{\theta}} g(\mu(\theta_j) - \mu(\theta))r_1(\widehat{\theta}_j,\theta)f(\theta)d\theta\frac{d\mu(\theta_j)}{d\theta_j} \qquad (10)$$

A sufficient (but not necessary) condition for (10) to be positive for all $\widehat{\theta},\theta$ is that $r_1(\widehat{\theta}_j,\theta)$ is positive for all θ. If this is the case, then the standard single-crossing condition is satisfied, and a separating contract exists.

Lemma 1 *Consider prize premium functions $r(\widehat{\theta},\theta) \geq 0$ defined on $[\underline{\theta},\overline{\theta}] * [\underline{\theta},\overline{\theta}]$ with $r_1(\widehat{\theta},\theta) \geq 0$ for all $\theta,\widehat{\theta}$. There then exists a loser prize function $W(\widehat{\theta})$ consistent with truth-telling.*

First-best efficiency requires that

$$V = C'(\mu_j; \theta_j)$$

and as before we refer to $\mu^*(\theta)$ as first-best investment which is strictly increasing in θ.

From the first order condition of the agent's maximization problem we can derive the following: if there exists a prize premium function $r(\theta_j, \theta)$ satisfying Lemma 1 and such that

$$\int_{\underline{\theta}}^{\overline{\theta}} g(\mu^*(\theta_j) - \mu^*(\theta))r(\theta_j, \theta)f(\theta)d\theta = C'(\mu^*(\theta_j); \theta_j) \equiv V \quad all\ \theta_j \tag{11}$$

the generalized contest yields first-best incentives.

The main result, Proposition 1 below, states that first-best efficiency is achievable in a contest. The mechanism is to offer prize premia functions $r(\theta_j, \theta)$ that are i) increasing in own announcement θ_j, and ii) decreasing in the opponent's announcement θ. The self-selected prize premium $r(\theta_j, \theta)$ then provides incentives that corresponds to $r^*(\theta_j)$ as defined by (4).

To conceptualize this result, consider the following situation: assume that the ability distribution is represented by a standard symmetric single-peaked distribution. Due to the low density of types in the low-ability tail, low-ability contestants have weak incentive power in a standard contest. To restore efficiency, their stake in the contest must be strengthened; in other words they must be provided with a larger prize premium. Since their ability is low and it is likely that they will lose, they must also be provided with a high losing prize. To prevent high-ability types from mimicking low-ability types, the contract designed for low-ability types yields a low prize premium if a contestant (who pretends to be of low-ability) beats a more skilled opponent. The point is that a low-ability type does not suffer much from a low prize premium conditional on beating a more skilled contestant, as it is unlikely that she will actually beat this superior opponent. A high-ability contestant that mimics lower-ability types, however, is "punished" in the sense that she can only obtain a high prize premium by facing an opponent who claims to be a low type, which again is highly unlikely. Observe that the expected prize premium of the low type, *conditional on winning,* exceeds its unconditional expectation in a kind of "reversed winner's curse." Hence low-ability types are induced to "bid" aggressively, without providing incentives for high types to mimic low types.

The main result is formulated in the following proposition

Proposition 1 *First-best incentives are achievable in the generalized contest scheme*

The proof and a generalization of the result to n contestants is given in the appendix. The proposition characterizes prize premium functions providing first-best incentives in a two-player contest. The result is easily generalized to a multi-agent setting; this generalization is presented in appendix.

Since the contest organizer's incentives are compatible with those of the social planner's, the efficient contest is also an optimal contest[13]. To obtain this result, it is crucial that contestants learn of their ability after entering the contest and paying the entry fee; hence rationality constraints must be specified ex ante. In the literature it is sometimes assumed that contestants know their own ability before they decide whether to participate in the contest, such as in Fullerton and McAfee (1999). The rationality constraint is thus type-specific; each type, knowing her own ability, must be at least as well-off participating in the contest as in her best alternative choice. This is referred to as ex post rationality constraints. With ex post rationality con-

[13] As discussed in Dye (1984), see also Mookherjee (1984) and Potters, Rockenbach, Sadrieh and van Damme (2004), contest schemes are vulnerable to collusion. As rewards are solely determined by rank, agents may reduce their effort proportionally, thus reducing effort cost, with no impact on gross rewards. A complete analysis of collusion requires a dynamic model, which goes beyond this analyis.

straints, the optimal contest design ceases to be socially efficient, as shown below.

Ex post rationality constraints can also be motivated by the problem of time inconsistency, a problem that is suppressed above. After agents have entered the contest and learned of their respective abilities, the principal has an incentive to adjust the prize structure in order to expropriate some of the information rent obtained by high types. However, in the long run she has an incentive to commit to not making any adjustments to the scheme, since perceptive contestants, realizing that adjustments may occur, will not be willing to pay as high an entry fee as they would if rent extraction were not an issue. Thus the model described above can be interpreted as a case in which the principal credibly commits not to adjust the contest design. The opposite case, in which the contest designer cannot credibly commit, is one possible interpretation of the model addressed in the next section.

5 Discussion

The optimal contest scheme, as derived above, maximizes social surplus. In this section I will address two further points. The first point regards the *rationality constraint.* Since the rationality constraint is specified as an *ex ante* condition, rent extraction is not an issue. The second point regards the specification of the objective function and constraints on the optimization problem. Since gross revenue is linear in effort, the objective function used above is consistent with a common specification in the contest literature, the maximization of total effort (see references below). However, the constraints on the optimization problem are often specified differently in the literature, for instance, in the form of various restrictions on the prize structure. This specification has direct consequences for the optimal design and, by extension, for the welfare implications of the contest.

5.1 Ex post rationality constraints and rent extraction

I will now derive the optimal contest design in the same setting as above, but instead assume that agents learn their ability before they enter the contest. Hence high-ability agents obtain an information rent. The optimal contest scheme now deviates from static first-best effort, since the prize scheme can be used as a device for rent extraction.

As above, expected social surplus, per player, is the difference between expected effort and expected cost:

$$S = \int_{\underline{\theta}}^{\overline{\theta}} \left[V\mu(\theta) - C(\mu(\theta), \theta) \right] f(\theta) d\theta$$

Expected profit can be written as,

$$\Pi = S - U = \int_{\underline{\theta}}^{\overline{\theta}} \left[V\mu(\theta) - C(\mu(\theta), \theta) - u(\theta) \right] f(\theta) d\theta$$

with individual rationality constraint

$$u(\theta) \geq \underline{u} \text{ } all \text{ } \theta$$

Observe the difference between this program's rationality constraint, which states that each type θ must obtain at least the utility level $\underline{u}$, and the formulation above, in which the expected utility obtained must be at least $\underline{u}$. Expected profit is maximized subject to the player's incentive compatibility constraint, a monotonicity condition on the effort function, and the player's participation constraint, respectively:

$$\begin{aligned} u'(\theta) &= -C_2'(\mu(\theta),\theta) > 0 \\ \mu'(\theta) &\geq 0 \\ u(\underline{\theta}) &= \underline{u} \end{aligned}$$

Suppressing the monotonicity constraint for a moment, the associated Hamiltonian is given by the following expression:

$$H = [V\mu - C(\mu,\theta) - u(\theta)]\, f(\theta) - \lambda C_2'(\mu,\theta)$$

First-order condition with respect to μ is,

$$V - C_1(\mu,\theta) = \lambda C_{21}''(\mu,\theta) \tag{12}$$

Since $\dot{\lambda} = -\frac{dH}{du}$ it follows that

$$\dot{\lambda} = f(\theta) \tag{13}$$

With no binding restriction on $u(\overline{\theta})$, we have $\lambda(\overline{\theta}) = 0$, thus from (13),

$\lambda(\theta) = F(\theta) - 1$. Inserted in (12) yields

$$V - C_1(\mu, \theta) = -(1 - F(\theta))C''_{21}(\mu, \theta)$$

thus $V = C_1(\mu(\overline{\theta}), \overline{\theta})$, which is the standard efficiency on the top result.

The monotonicity condition $\mu'(\theta) \geq 0$ is satisfied under weak conditions on the cost function.[14] Since $C''_{21}(\mu, \theta) < 0$, it follows that $V > C_1(\mu, \theta)$ all $\theta < \overline{\theta}$. Thus effort is sub-optimal, $\mu(\theta) < \mu^*(\theta)$.

The proof of existence of a prize premium function supporting $\mu(\theta)$ corresponds to the proof of Proposition 1. Thus, the optimal prize premium gives for all $\theta < \overline{\theta}$ a strictly lower incentive power than does the first best premium $r^*(\theta)$ as defined by (4). In other words, rent is extracted by decreasing the prize premium, providing agents with sub-optimal effort incentives.

Proposition 2 *The optimal contest with heterogeneous agents yields effort* $\mu(\overline{\theta}) = \mu^*(\overline{\theta}), \;\; \mu(\theta) < \mu^*(\theta)$ *all* $\theta < \overline{\theta}$

The result that equilibrium effort is *below* static first-best deserves further comment. This seemingly contradicts some of the equilibria described in the

[14] For instance, a sufficient condition for monotonicity is a quadratic cost function, in which case all third derivatives of the $C(\cdot)$ function are zero. Also, the common specification in which effort cost is inversely related to ability, $C(\cdot) = \mu^2/\theta$, yields a monotone response.

contest literature, where agents are induced to exert effort *above* static first-best. As will be shown, this ostensible inconsistency is simply a result of different assumptions with regard to the restrictions on the contract set.

5.2 Effort maximizing contests

In the model above it is assumed that the principal maximizes net surplus, i.e. the expected value of total output net of total prize expenses, conditional on agents' rationality constraints. In the contest literature, it is commonly assumed that the principal's goal is to maximize expected total effort subject to constraints on the prize structure; see for example Moldovanu and Sela (2001), Gradstein and Konrad (1999), and Glazer and Hassin (1988).[15]

For clarification, consider the following specification of the optimization problem: the principal maximizes expected total effort conditional on a given prize budget and on the assumption that each prize is non-negative.[16] Agents' effort levels can then be derived from the solution of this program. However, the welfare implications of the program are ambiguous. Cost efficiency re-

[15]The specification of the surplus function is more of a semantic question. Since gross surplus in my model is linear in the sum of effort, maximizing net surplus corresponds to maximizing expected total effort net of total prize expenses. However, the differences in the specifications of the constraints on the optimization problem impact the welfare analysis, as will be explored below.

[16]This is the formulation used in all three references above.

quires that marginal effort costs across types be aligned, and in first-best, marginal effort costs are equal to marginal social benefit. However, when the principal optimizes under the constraint that all prizes are non-negative and with a given prize budget, optimal design depends on the prize structure's marginal impact on effort; thus on the derivative of marginal cost, instead of the level of marginal cost. In principle, this could go either way. As the literature has established, excessive effort may occur in such equilibrium, an inefficiency reflecting this specific constraint on the optimization problem.[17]

Let us derive the optimal prize menu with a specification that corresponds to the common formulation in the contest literature: the principal maximizes the expected sum of effort conditional on i) an upward constraint on prize expenses, and ii) a non-negativity constraint on prizes. With a prize menu, there are two alternative interpretations of the latter constraint. The first possibility is to attach a non-negativity restriction on *realized* prizes. That rules out a negative base wage W. The second specification is a non-negativity constraint on the expected prize obtained by each type θ. As long as the win probability is strictly positive, this allows for a negative W. I explore both alternatives. Regarding the restrictions on total expenses, I choose

[17] As pointed out by O'Keeffe, Viscusi and Zeckhauser (1984), contests eliciting excessive effort will, under perfect competition, be driven out of existence.

the formulation that *expected prize expenses* are upwardly constrained[18].

Consider the following program: the principal maximizes the value of expected total effort subject to i) a limit $\overline{r}$ on expected expenses, ii) a non-negativity constraint on realized prizes, and iii) the incentive constraint.

$$\text{maximize} \int_{\underline{\theta}}^{\overline{\theta}} \mu(\theta) f(\theta) d\theta \text{ s.t.}$$

$$i) \;\; E[r(\theta_j, \theta) + W(\theta_j) + W(\theta)] \leq \overline{r},$$

$$ii) \;\; W(\theta) \geq 0 \text{ all } \theta$$

$$iii) \;\; u'(\theta) = -C_2'(\mu(\theta), \theta)$$

To simplify, let prize premium r depend only on the opponent's report, hence the program satisfies the requirement of weak truth-telling. As before, it is simple enough to modify the scheme to satisfy the condition of strict truth-telling.[19]

[18] The alternative specification, that realized prize expenses are constrained, does not provide any additional insight, but adds to the complexity.

[19] However, since incentive power is independent of the base wage W, and since there is a constraint on total expenses, a positive base wage W is costly, given that a reduction in r is required in order to comply with the constraint. Hence, stipulating that W be equal to 0 yields the unique optimal scheme. Technically, if strict truth-telling were to be required - the optimum would be represented by the limit as W approaches zero.

The agent's first order condition for effort is expressed as,

$$\int_{\underline{\theta}}^{\overline{\theta}} g(\mu(\theta_j) - \mu(\theta))r(\theta)f(\theta)d\theta - C_1'(\mu(\theta_j), \theta_j) = 0 \tag{14}$$

where $r(\theta_j, \theta)$ is simplified to $r(\theta)$ since it only depends on the opponent's report. Since effort $\mu(\theta)$ is increasing in type, the winner's prize premium $r(\theta)$ is determined by the loser's type θ. The CDF of the second-highest ability is $F_2(\theta) = 2(1 - F(\theta))F(\theta) + F(\theta)^2$ with associated density $f_2(\theta) = 2(1 - F(\theta))f(\theta)$. Thus expected prize expenses can be expressed as,

$$Er = \int_{\underline{\theta}}^{\overline{\theta}} r(\theta)f_2(\theta)d\theta$$

The contest organizer maximizes expected effort $E\mu_j = \int_{\underline{\theta}}^{\overline{\theta}} \mu(\theta_j)f(\theta_j)d\theta_j$ with respect to the prize menu $r(\theta)$ subject to $Er \leq \overline{r}$. The associated Langrangian is expressed,

$$L(r, \lambda) = \int_{\underline{\theta}}^{\overline{\theta}} \mu(\theta_j)f(\theta_j)d\theta_j - \lambda \left[\int_{\underline{\theta}}^{\overline{\theta}} r(\theta)f_2(\theta)d\theta - \overline{r} \right]$$

Point-wize maximization yields,

$$\int_{\underline{\theta}}^{\overline{\theta}} \frac{d\mu(\theta_j; r)}{dr(\theta)} f(\theta_j) d\theta_j = \lambda f_2(\theta)$$

or,

$$\int_{\underline{\theta}}^{\overline{\theta}} \frac{d\mu(\theta_j; r)}{dr(\theta)} f(\theta_j) d\theta_j = 2\lambda(1 - F(\theta)) f(\theta) \tag{15}$$

The left hand side of (15) captures the effect the prize premium has on effort best responses. As pointed out above, since the best response aligns marginal cost with marginal utility, the prize premium's marginal impact on best response thus depends on the derivative of marginal cost and the derivative of marginal utility. The right hand side captures the cost side of increased prize premia. Due to the term $1 - F(\theta)$, providing high types with high incentive power does not contribute much to expected expenses, as it is unlikely that the loser's type, which determines the winner's prize premium, is also high. Thus, high types are provided with, relative to first-best incentives, higher incentive power than are low types. Furthermore, as θ approaches $\overline{\theta}$, $1 - F(\theta)$ approaches zero, thus high types are induced to exert excessive effort.

Let me add one comment regarding this result. As shown above, without restrictions on the prize scheme, maximizing effort, subject to standard incentives and rationality constraints, yields the standard result with efficiency on top and with sub-optimal effort from all other types. In contrast, when non-negativity restrictions on prizes are introduced, high-ability agents typically exert excessive effort. This can be explained as follows: the non-negativity constraint on prizes rules out the following redesign of the contract for high-ability contestants: reduce the prize premium r to obtain efficiency on the top and extract surplus through a negative base wage W. Thus the non-negativity restriction on prizes is essential. This will become clear in the discussion below where the non-negativity constraints on prizes are relaxed.

We replace the condition that each realized prize should be non-negative with the condition that expected prizes are non-negative, that is, in equilibrium:

$$E\left[W(\theta_j) + \int_{\underline{\theta}}^{\overline{\theta}} G(\mu(\theta_j) - \mu(\theta))r(\theta)f(\theta)rd\theta\right] \geq 0 \quad \text{all } \theta_j \tag{16}$$

Clearly, this allows the principal to set W below zero, as long as it is compensated by a sufficiently large expected prize premium r. Then it follows: since the win probability $\int_{\underline{\theta}}^{\overline{\theta}} G(\mu(\theta_j) - \mu(\theta))f(\theta)rd\theta$ is strictly positive for all θ_j, (16) implies that there is no upper limit on the level of $r(\theta)$ for any expected prize fund. Thus $r(\theta)$ can be increased, and total prize expenses can be kept constant through a downward adjustment of $W(\theta)$. Since effort is strictly increasing in $r(\theta)$, it follows that the maximization problem is unbounded. Obviously, the missing component here is agents' rationality constraints. In other words, with a proper re-specification, the rationality constraints strictly bind, whereas the non-negativity constraint on expected prize expenses do not. Including the rationality constraints, it is easy to demonstrate that an effort maximizing contest with given expected prize expenses is equivalent to the efficient contest analyzed above. Due to rent extraction, effort is below static efficiency for all types, except for the highest type who exerts first-best effort.

6 Concluding remarks

This paper has demonstrated how a generalized contest can be designed to support efficient allocation under dual information asymmetry, situations in which ability is private information and output only observable with noise. High-ability contestants' incentives are restored by providing those individuals with a large upside, a high prize premium and a low loser's prize. Low-ability contestants' incentives are restored by awarding them a high prize premium for winning, *conditional* on being challenged by other low-ability opponents. The latter restriction ensures that the contract is unattractive to high-ability types.

This sorting problem is common and well-established in incentive contract literature. For clarification, consider the following classic information problem within incentive contract design: when creating incentive schemes, designers often aim to enhance the incentive power *locally,* for instance by providing the agent with a bonus for accomplishing a task or fulfilling a target. Clearly, setting the target too low or too high yields poor incentives, as the bonus will be either almost certain or unattainable. Hence, adjusting the target constitutes a classic information problem when ability is private information. To conceptualize this, consider the piecewise linear "kinked" incentive scheme analyzed in Weitsman (1976), see also Holmström (1982), which unfolds as follows: initially a single agent is presented with a tentative reward scheme consisting of a strictly increasing linear function of observed output. Thereafter, the agent selects her final reward scheme, which is piecewise linear with one kink. The essential point is that this final reward function is strictly below the tentative reward function everywhere except one single point — the self-selected kink. Clearly the agent is best-off choosing a kink that corresponds exactly with whichever output level is optimal, given her inherent ability and given the initial tentative scheme. Thus, the sorting condition is satisfied since the agent, relative to the tentative scheme, is punished by choosing a low kink, i.e. by mimicking low types, as well as by choosing a high kink. i.e. mimicking high types.

The self-selected "kink" as a sorting device corresponds logically to the mechanism that yields self-sorting in the optimal contest derived in this paper, in which contestants choose from a menu of prizes. Since the principal only has access to ordinal information, the reward can neither be conditioned on observed output nor on the bid (investment) itself. Yet the reward can be conditioned on the opponents' types, as this information is revealed through their self-selection of contest prizes. Mimicking inferior types is thereby avoided, as shown, by "punishing" agents for beating superior opponents.

If agents know their ability before entering the contest, rent extraction becomes an issue. In this situation, the principal's optimal scheme yields effort below the socially efficient level for all types except the highest, often referred to as "efficiency on the top". This conclusion seemingly contradicts the conclusion in the contest literature that effort-maximizing contests may yield excessive effort. However, this discrepancy results from the different specifications of the constraints imposed upon the prize structure.

7 Appendix: proof Proposition 1

From (11) and Lemma 1 we know that a prize premium function $r(\theta_j, \theta)$ that satisfies

$$\int_{\underline{\theta}}^{\overline{\theta}} g(\mu^*(\theta_j) - \mu^*(\theta)) r(\theta_j, \theta) f(\theta) d\theta = V \ all \ \ \theta_j \epsilon [\underline{\theta}, \overline{\theta}] \tag{17}$$

and where $r(\theta_j, \theta)$ is monotonically increasing in θ_j, provides efficient effort incentives.

It is convenient to use μ as the integration variable instead of θ. Let $\theta^*(\mu)$ denote the inverse of the first-best efficiency effort function $\mu^*(\theta)$ and apply the following transformation of (17)

$$\begin{aligned} \int_{\underline{\mu}}^{\overline{\mu}} g(\mu_j - \mu) H(\mu_j, \mu) d\mu \quad &: \quad = V(\mu_j) = V \ all \ \ \mu_j \epsilon [\underline{\mu}, \overline{\mu}] \\ \text{where } H(\mu_j, \mu) \quad &\equiv \quad r(\theta(\mu_j), \theta(\mu)) f(\theta(\mu)) \frac{d\theta}{d\mu} \end{aligned} \tag{18}$$

where $\overline{\mu} = \mu^*(\overline{\theta})$ and $\underline{\mu} = \mu^*(\underline{\theta})$. Sorting requires that $H(\mu_j, \mu)$ is increasing in μ_j.

The proof goes as follows: I construct a function $H(\mu_j, \mu)$ that satisfies (18) and the sorting condition for all $\mu_j \epsilon [\underline{\mu} + \varepsilon, \overline{\mu}]$, where ε is a small number. Then, I let ε be arbitrarily small. In the small interval $\mu_j \epsilon [\underline{\mu}, \underline{\mu} + \varepsilon)$ there will be a deviation from first best of size $|V(\mu_j) - V|$. However, due to continuity, the deviation is a small number. Furthermore, as the probability that one of the contestants' types belong to the segment $[\underline{\mu}, \underline{\mu} + \varepsilon)$ is a small number as well, the expected deviation from first best is of second order.[20]

Let $\mathbf{h}$ be a number determined as follows

$$\int_{\underline{\mu}}^{\underline{\mu}+2\varepsilon} g(\underline{\mu} + \varepsilon - \mu)\mathbf{h} d\mu = V \quad \Rightarrow \quad \mathbf{h} = \frac{V}{G(\varepsilon) - G(-\varepsilon)} \tag{19}$$

For all $\mu_j \geq \underline{\mu} + \varepsilon$ let $k(\mu_j)$ be implicitly determined by,

$$\int_{\underline{\mu}}^{\underline{\mu}+k(\mu_j)} g(\mu_j - \mu)\mathbf{h} d\mu = V \;\Rightarrow\; \mathbf{h} = \frac{V}{G(\mu_j - \underline{\mu}) - G(\mu_j - k(\mu_j) - \underline{\mu})} \tag{20}$$

[20] Since (17) is conditional on opponents exerting first best effort, the deviation from first best for types $\mu_j \epsilon [\underline{\mu}, \underline{\mu} + \varepsilon)$ will slightly distort the effort choice of all types. This distortion is also of second order for small values of ε.

Observe that $k(\underline{\mu}+\varepsilon)=2\varepsilon$. Set

$$H(\mu_j,\mu)=\left\{\begin{array}{c}\mathbf{h}\ \ if\ \ \mu\epsilon\left[\underline{\mu},\underline{\mu}+k(\mu_j)\right]\\ \\ 0\ otherwize\end{array}\right\} \tag{21}$$

Inserting (21) in (18) shows that the first order condition for first best effort is satisfied. In addition we need to check that (21) satisfies the sorting condition. We see that $H(\mu_j,\mu)$ is monotonically increasing in μ_j if $k(\mu_j)$ is non-decreasing. From (19) and (20) we have that $G(\mu_j-\underline{\mu})-G(\mu_j-k(\mu_j)-\underline{\mu})=G(\varepsilon)-G(-\varepsilon)$ thus

$$k'(\mu_j)=1-\frac{g(\mu_j-\underline{\mu})}{g(\mu_j-k(\mu_j)-\underline{\mu})}\geq 0$$

Clearly $k'(\underline{\mu}+\varepsilon)\geq 0$ as $k(\underline{\mu}+\varepsilon)=2\varepsilon$. Furthermore, since $d\left(\mu_j-k(\mu_j)\right)/d\mu_j<1$ and since $k(\mu_j)$ is non-negative, it follows that $|\mu_j-k(\mu_j)-\underline{\mu}|\leq|\mu_j-\underline{\mu}|$ for all μ_j. Thus (21) satisfies the sorting condition.

It remains to generalize the proof to n contestants. Since the probability of winning the contest increases with ability, separation is feasible in a contest that remunerates the overall winner with a prize premium and pays the remaining contestants their respective loser prizes[21]. Let me provide an outline of this procedure. Denote by $\mathbf{F}(\theta)$ the probability that the highest competing type is θ, and by $\mathbf{G}(\mu_j - \mu^*_{(-)}(\theta))$ the probability that the contestant beats all of her opponents, with types drawn below θ according to the type distribution $F(.)$, truncated at θ. Then, the utility of contestant j can be expressed as,

$$U(\theta_j, \widehat{\theta}_j) = \max_{\mu_j} \left[W_2(\widehat{\theta}_j) + \int_{\underline{\theta}}^{\overline{\theta}} \mathbf{G}(\mu_j - \mu^*_{(-)}(\theta)) r(\widehat{\theta}_j, \theta) d\mathbf{F}(\theta) - C(\mu_j; \theta_j) \right]$$

with the first-order condition (in a truth-telling equilibrium),

$$\int_{\underline{\theta}}^{\overline{\theta}} \mathbf{g}(\mu^*(\theta_j) - \mu^*_{(-)}(\theta)) r(\theta_j, \theta) d\mathbf{F}(\theta) d\theta = C'(\mu^*(\theta_j); \theta_j)$$

(in which $\mathbf{g}$ denotes the density of $\mathbf{G}$). The proof of Proposition 1 can now be replicated.

[21]Remunerating the overall winner is sufficient.

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